S_4 Flavor Symmetry Embedded into SU(3) and Lepton Masses and Mixing

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Abstract

Based on the assumption that an S_4 flavor symmetry is embedded into SU(3), a lepton mass matrix model is investigated. A Froggatt-Nielsen type model is assumed, and the flavor structures of the masses and mixing are caused by VEVs of $SU(2)_L$-singlet scalars ϕ_u and ϕ_d which are nonets ($\mathbf{8}+\mathbf{1}$) of the SU(3) flavor symmetry, and which are broken into $\mathbf{2}+\mathbf{3}+\mathbf{3}'$ and $\mathbf{1}$ of S_4. If we require the invariance under the transformation $(\phi^{(8)}, \phi^{(1)}) \to (-\phi^{(8)}, +\phi^{(1)})$ for the superpotential of the nonet field $\phi^{(8+1)}$, the model leads to a beautiful relation for the charged lepton masses. The observed tribimaximal neutrino mixing is understood by assuming two SU(3) singlet right-handed neutrinos $\nu_R^{(\pm)}$ and an SU(3) triplet scalar χ.

1 Introduction

The observed mass spectra and mixings of the fundamental particles will provide promising clues to unified understanding of the quarks and leptons. Especially, in the lepton sector, the following characteristic features have been observed [1]:
(i) The observed charged lepton masses (m_e, m_μ, m_τ) satisfy the relation [2, 3]

$$m_e + m_\mu + m_\tau = \frac{2}{3}(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2, \tag{1.1}$$

with remarkable precision;
(ii) The observed neutrino mixing U_ν is approximately given by the so-called tribimaximal mixing [4]

$$U_{TB} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{1.2}$$

Such characteristic features have not been seen in the quark sector. For example, the mixing form (1.2) suggests that the mixing can be described by Clebsh-Gordan-like coefficients, while, for the Cabibbo-Kobayashi-Maskawa mixing in the quark sector, such a characteristic feature has not been seen, although we have known some relations among the mixing angles and quark mass ratios. Therefore, for a start, in the present paper, we investigate the lepton masses and mixings.

In order to understand the relation (1.1), for example, we assume that there are three scalars ϕ_i $(i = 1, 2, 3)$, and the values of the charged lepton masses m_{ei} are proportional to the square of the vacuum expectation values (VEVs) $v_i = \langle\phi_i\rangle$ of the scalars ϕ_i, $m_{ei} = kv_i^2$ (in the

Ref.[3, 5, 6], for instance, a seesaw type model $(M_e)_{ij} = \delta_{ij} v_i (M_E)^{-1} v_j$ has been assumed). We define singlet ϕ_σ and doublet (ϕ_π, ϕ_η) of a permutation symmetry S_3 [7] by

$$\begin{pmatrix} \phi_\pi \\ \phi_\eta \\ \phi_\sigma \end{pmatrix} = \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}, \tag{1.3}$$

from the three objects (ϕ_1, ϕ_2, ϕ_3), and we consider the following S_3 invariant scalar potential $V(\phi)$ [3, 8, 9]:

$$V(\phi) = m^2(\phi_\pi^2 + \phi_\eta^2 + \phi_\sigma^2) + \lambda_1(\phi_\pi^2 + \phi_\eta^2 + \phi_\sigma^2)^2 + \lambda_2 \phi_\sigma^2(\phi_\pi^2 + \phi_\eta^2). \tag{1.4}$$

The minimizing condition of the potential (1.4) leads to the relation

$$v_\pi^2 + v_\eta^2 = v_\sigma^2. \tag{1.5}$$

The relation (1.5) means

$$v_1^2 + v_2^2 + v_3^2 = \frac{2}{3}(v_1 + v_2 + v_3)^2, \tag{1.6}$$

because

$$v_1^2 + v_2^2 + v_3^2 = v_\pi^2 + v_\eta^2 + v_\sigma^2 = 2v_\sigma^2 = 2\left(\frac{v_1 + v_2 + v_3}{\sqrt{3}}\right)^2. \tag{1.7}$$

Therefore, we can obtain the mass relation (1.1). Here, note that although the scalar potential (1.4) is invariant under the S_3 symmetry, but it is not a general one of the S_3 invariant form. As pointed out in Ref. [9], the scalar potential with a general form cannot lead to the relation (1.5). For the derivation of the VEV relation (1.5), it is essential to choose the specific form (1.4) of the S_3 invariant terms. Similar formulation is also possible for other discrete symmetries A_4 [10] and S_4 (see below). However, in such a symmetry, we still need an additional specific selection rule. What is the meaning of such a specific selection? In the present paper, we investigate this problem by assuming that the S_4 flavor symmetry is embedded into SU(3).

Recently, a superpotential which leads to the relation (1.5) has proposed by Ma [11] on the basis of a symmetry $\Sigma(81)$. Stimulated by the Ma's idea, the author [10] has also investigated a similar superpotential on the basis of a symmetry A_4. Here, based on an S_4 flavor symmetry instead of the A_4 symmetry, let us review the superpotential W which gives the relation (1.5). We denote singlet and doublet of S_4 as ϕ_σ and $\phi_D = (\phi_\pi, \phi_\eta)^T$, respectively, as well as those in S_3. In order to write the superpotential for the scalar fields ϕ_σ and doublet ϕ_D of S_4, we put the following phenomenological rule [10]: the field ϕ_a $(a = \sigma, D)$ to the power nth, $(\phi_a)^n$ $(n = 1, 2, 3)$, appears always accompanied with the factor $1/n!$ in the superpotential W. Under this phenomenological rule, we can uniquely write the superpotential of ϕ_σ and ϕ_D as

$$W(\phi) = \frac{1}{2!} m \left(\phi_\sigma^2 + \phi_D^T \phi_D\right) + \lambda \left(\frac{1}{2!}\phi_\sigma \phi_D^T \phi_D + \frac{1}{3!}\phi_\sigma^3\right)$$

$$= \frac{1}{2}m\left(\phi_\sigma^2+\phi_\pi^2+\phi_\eta^2\right)+\frac{1}{2}\lambda\left[(\phi_\pi^2+\phi_\eta^2)\phi_\sigma+\frac{1}{3}\phi_\sigma^3\right]. \tag{1.8}$$

The potential (1.8) can also lead the relation (1.5). What is the meaning of this phenomenological rule?

On the other hand, we have to consider a mechanism which yields the charged lepton masses $m_{ei} \propto v_i^2$, i.e. the effective Hamiltonian for the charged lepton sector

$$H_e^{eff} = \left[\bar{e}_{L1}(\phi_1)^2 e_{R1} + \bar{e}_{L2}(\phi_2)^2 e_{R2} + \bar{e}_{L3}(\phi_3)^2 e_{R3}\right]. \tag{1.9}$$

We will propose a Froggatt-Nielsen type model [12], $H_e^{eff} = (\bar{\ell}_L H_L^d \phi\phi e_R)$ in Sec.4.

Now, let us return the topic of the tribimaximal mixing. From the definition (1.2), we can denote the fields (ψ_1, ψ_2, ψ_3) as

$$\begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} = U_{TB} \begin{pmatrix} \psi_\eta \\ \psi_\sigma \\ \psi_\pi \end{pmatrix}. \tag{1.10}$$

The observed neutrino mixing (1.2) means that when the mass eigenstates of the charged leptons are given by the (ψ_1, ψ_2, ψ_3) basis, the mass eigenstates of the neutrinos are given by the $(\psi_\eta, \psi_\sigma, \psi_\pi)$ basis. Therefore, the problem is to find a model where the charged lepton mass eigenstates are (e_1, e_2, e_3), while the neutrino mass eigenstates are given by $(\nu_\eta, \nu_\sigma, \nu_\pi)$ with the masse hierarchy $m_\eta^2 < m_\sigma^2 \ll m_\pi^2$ (or $m_\pi^2 \ll m_\eta^2 < m_\sigma^2$). In the present paper, we will investigate such a model based on an S$_4$ model. Here, note that the fermions (ψ_1, ψ_2, ψ_3) is a triplet of S$_4$, but the basis $(\psi_\eta, \psi_\sigma, \psi_\pi)$ is not in any irreducible representations of S$_4$, while the scalar (ϕ_1, ϕ_2, ϕ_3) is not irreducible representation of S$_4$, but (ϕ_π, ϕ_η) and ϕ_σ are doublet and singlet of S$_4$.

Thus, the characteristic features (1.1) and (1.2) in the lepton sector may be understood from the language of S$_4$ (also S$_3$ or A$_4$). However, as seen from the above review, the characteristic features (1.1) and (1.2) cannot be understood from the S$_4$ symmetry only. We need some additional assumptions. In this paper, we will investigate these problems under an assumption that the present S$_4$ symmetry is embedded into an SU(3) symmetry [13]. In the next section, the singlet ϕ_σ and doublet (ϕ_π, ϕ_η) will be understood as members of a nonet scalar ϕ [**1**+**8** of SU(3)], and the VEV relation (1.5) will be derived by requiring that $W(\phi)$ is invariant under a Z$_2$ symmetry.

We know that the three masses in any sectors of quarks and leptons are completly different among them. Therefore, if we assume a flavor symmetry, the symmetry must finally be broken completely. Usually, a relation which we derive in the exact symmetry limit is only approximately satisfied under the symmetry breaking. Although we derive the VEV relation (1.5) under the S$_4$ symmetry, the problem is whether the VEV relation (1.5) which is obtained under the S$_4$ symmetry is spoiled or not when we introduce such a symmetry breaking. In Sec.3, we will demonstrate that such a symmetry breaking term without spoiling the relation (1.5) is indeed possible.

In Sec.4, in order to give the charged lepton masses and tribimaximal neutrino mixing, we will discuss the effective Hamiltonian by assuming an Froggatt-Nelsen [12] type model. Finally, Sec.5 will be devoted to the summary and concluding remarks.

2 VEVs of SU(3) nonet scalars

The goal in the present section is to obtain the VEV relation (1.6) [i.e. (1.5)]. As seen in the previous section, in order to obtain the desirable results (1.5), we need assume an equal weight between the doublet and singlet terms of S_4. In the present paper, we assume that the S_4 symmetry is embedded into an SU(3) symmetry. The doublet (ϕ_π, ϕ_η) and singlet ϕ_σ of S_4 are embedded in the **6** and $(\mathbf{8}+\mathbf{1})$ of SU(3) [13]. In the present paper, we assume that the doublet (ϕ_π, ϕ_η) and singlet ϕ_σ originate in SU(3) octet and singlet, respectively. The essential assumption in the present paper is that the fields ϕ_u and ϕ_d always appear in the theory with the form of the nonet of U(3):

$$\phi = \begin{pmatrix} \phi_1^1 & \phi_1^2 & \phi_1^3 \\ \phi_2^1 & \phi_2^2 & \phi_2^3 \\ \phi_3^1 & \phi_3^2 & \phi_3^3 \end{pmatrix}, \tag{2.1}$$

where

$$\begin{aligned} \phi_1^1 &= \tfrac{1}{\sqrt{3}}\phi_\sigma + \tfrac{2}{\sqrt{6}}\phi_\eta, \\ \phi_2^2 &= \tfrac{1}{\sqrt{3}}\phi_\sigma - \tfrac{1}{\sqrt{6}}\phi_\eta - \tfrac{1}{\sqrt{2}}\phi_\pi, \\ \phi_3^3 &= \tfrac{1}{\sqrt{3}}\phi_\sigma - \tfrac{1}{\sqrt{6}}\phi_\eta + \tfrac{1}{\sqrt{2}}\phi_\pi, \end{aligned} \tag{2.2}$$

and the index f $(f = u, d)$ has been dropped.

The outline to obtain the superpotential form (1.8) in the present scenairo is as follows: The SU(3) invariant superpotential for the nonet fields ϕ_f $(f = u, d)$ are given by

$$W(\phi_f) = \frac{1}{2} m_f \mathrm{Tr}(\phi_f \phi_f) + \frac{1}{2\sqrt{3}} \lambda_f \mathrm{Tr}(\phi_f \phi_f \phi_f). \tag{2.3}$$

Since, in the next section, we want to assign chages $+1$ and -1 of a Z_3 symmetry to the fields ϕ_u and ϕ_d, respectively, we also assign the Z_3 charges $+1$ and -1 to the mass parameters m_u and m_d in Eq.(2.3), respectively. However, since we do not consider a mass term $\mathrm{Tr}(\phi_u \phi_d)$, we do not consider a mass parameter with the Z_3 charge zero. Hereafter, in the present section, for convenience, we will drop the index f, since the cross terms between ϕ_u and ϕ_d do not appear. In the superpotential (2.3), although the term $\mathrm{Tr}(\phi\phi)$ gives the desirable term $\phi_\pi^2 + \phi_\eta^2 + \phi_\sigma^2 + \cdots$ of S_4, the cubic term $\mathrm{Tr}(\phi\phi\phi)$ gives

$$\mathrm{Tr}(\phi\phi\phi) = \sqrt{3}\left[\frac{1}{\sqrt{2}}\left(-\phi_\pi^2 + \frac{1}{3}\phi_\eta^2\right)\phi_\eta + (\phi_\pi^2 + \phi_\eta^2)\phi_\sigma + \frac{1}{3}\phi_\sigma^3\right] + \cdots, \tag{2.4}$$

where the terms "$\cdots$" denote terms which include **3** and **3′** of the subgroup S_4. Therefore, the potential (2.3) cannot give the relation (1.5). We must drop the first term in the cubic terms

(2.4). For this purpose, we introduce a Z_2 symmetry, and we assign the Z_2 parities -1 and $+1$ (the Z_2 charge $+1$ and 0) for the octet part $\phi^{(8)}$ and singlet part $\phi^{(1)}$ of the nonet field ϕ, respectively. The symmetry Z_2 breaks U(3) into SU(3). (In other words, in the present model, the flavor symmetry U(3) is explicitly broken from the begining by the Z_2 symmetry.) Under the requirement of the Z_2 invariance, i.e. the invariance under the transformation

$$(\phi^{(8)}, \phi^{(1)}) \to (-\phi^{(8)}, +\phi^{(1)}), \tag{2.5}$$

the terms $\mathrm{Tr}(\phi^{(8)}\phi^{(8)}\phi^{(8)})$, i.e. $(-\phi_\pi^2 + (1/3)\phi_\eta^2)\phi_\eta + \cdots$, are forbidden. Thus, the superpotential (2.3) with the Z_2 invariance leads to

$$\begin{aligned} W(\phi) &= \frac{1}{2}m\left[\mathrm{Tr}(\phi^{(8)}\phi^{(8)}) + \phi_\sigma^2\right] + \frac{1}{2}\lambda\phi_\sigma\left[\mathrm{Tr}(\phi^{(8)}\phi^{(8)}) + \frac{1}{3}\phi_\sigma^2\right] \\ &= \frac{1}{2}m\left(\phi_\sigma^2 + \phi_\pi^2 + \phi_\eta^2\right) + \frac{1}{2}\lambda\left[(\phi_\pi^2 + \phi_\eta^2)\phi_\sigma + \frac{1}{3}\phi_\sigma^3\right] + \cdots . \end{aligned} \tag{2.6}$$

The form (2.6) is just identical with (1.8) except for the "$\cdots$" terms. As we show below, the potential (2.6) can give the desirable VEV relation (1.5) together with $\langle\phi_i^j\rangle = 0$ $(i \neq j)$.

From the superpotential (2.6) with the Z_2 invariance, we obtain the VEV relation (1.5) as follows: From the condition

$$\frac{\partial W}{\partial(\phi^{(8)})_i^j} = m(\phi^{(8)})_j^i + \lambda\phi_\sigma(\phi^{(8)})_j^i = 0, \tag{2.7}$$

we obtain

$$m + \lambda\phi_\sigma = 0, \tag{2.8}$$

for $(\phi^{(8)})_i^j \neq 0$. By eliminating m from Eq.(2.8) and the condition

$$\frac{\partial W}{\partial\phi_\sigma} = m\phi_\sigma + \frac{1}{2}\lambda\left[\mathrm{Tr}(\phi^{(8)}\phi^{(8)}) + \phi_\sigma^2\right] = 0, \tag{2.9}$$

we obtain the relation

$$\phi_\sigma^2 = \mathrm{Tr}(\phi^{(8)}\phi^{(8)}) = \phi_\pi^2 + \phi_\eta^2 + \cdots , \tag{2.10}$$

where "$\cdots$" denotes the contributions of **3** and **3**$'$ of S_4.

The result (2.10) is still not our goal, because the relation contains the VEVs of the **3** and **3**$'$ of S_4. So far, we have not discussed the splitting among the S_4 multiplets. Now, we bring a soft symmetry breaking of SU(3) into S_4 with an infinitesimal parameter ε into the mass term of $W(\phi)$ as

$$\mathrm{Tr}(\phi^{(8)}\phi^{(8)}) \Rightarrow \phi_\pi\phi_\pi + \phi_\eta\phi_\eta + (1+\varepsilon)\sum_{i\neq j}(\phi^{(8)})_i^j(\phi^{(8)})_j^i, \tag{2.11}$$

by hand. (At present, we do not refer the origin of the symmetry breaking. The SU(3) flavor symmetry is explicitly (not spontaneously) broken with the order of ε.) Recall that when we

obtain the relation (2.8), we have assumed $(\phi^{(8)})_i^j \neq 0$. Now, the conditions (2.7) are modified into the folloing conditons:

$$[(1+\varepsilon)m + \lambda\phi_\sigma]\,(\phi^{(8)})_j^i = 0 \quad (i \neq j), \tag{2.12}$$

$$(m + \lambda\phi_\sigma)\,\phi_a = 0 \quad (a = \pi, \eta), \tag{2.13}$$

Thefore, we must take either $(\phi^{(8)})_j^i = 0$ $(i \neq j)$ or $\phi_a = 0$ $(a = \pi, \eta)$ for $\varepsilon \neq 0$. When we choose the solution

$$\langle(\phi^{(8)})_j^i\rangle = 0 \quad (i \neq j), \tag{2.14}$$

we can obtain the desiarable relation (1.5). (However, it is possible that we can also take another solution with $\phi_\pi = \phi_\eta = 0$ and $(\phi^{(8)})_j^i \neq 0$. The VEV solutions are not unique. The result (1.5) is merely one of the possible solutions.)

Thus, we have obtained not only the desirable VEV relation (1.5), but also the results (2.14). It should be worthwhile noticing that if we have assume the superpotential (2.3) without requiring the Z_2 invariance, we could obtain neither (1.5) nor (2.14).

3 Superpotential with symmetry breaking

Since we know that the three masses in any sectors of quarks and leptons are completely different among them, we must consider that any flavor symmetry which we introduced should finally be broken completely. Although the superpotential (1.8) can give the VEV relation (1.5), it cannot fix the ratio v_π/v_η. In order to fix the ratio v_π/v_η, we consider the existence of an S_4 symmetry breaking term W_{SB}. Then, the problem is whether the VEV relation (1.5) which has been obtained under the S_4 symmetry which is embedded into SU(3) is spoiled or not by introducing such a symmetry breaking, because, usually, a relation which we have derived under an exact symmetry is only approximately satisfied under the symmetry breaking. In the present section, we will demonstrate that such a symmetry breaking term without spoiling the relation (1.5) is indeed possible.

We consider that the S_4 invariant superpotential (1.8) is softly broken. Since we want $v_\pi/v_\eta \neq 1$, the breaking should appear in the doublet part of S_4. In order to express the S_4 symmetry breaking term explicitly, we define the following symmetry breaking parameters $B^{(8)}$ and $B^{(1)}$ with 3×3 matrix forms,

$$\begin{aligned} B^{(8)} &= \mathrm{diag}\left(\tfrac{2}{\sqrt{6}}b_\eta, -\tfrac{1}{\sqrt{6}}b_\eta - \tfrac{1}{\sqrt{2}}b_\pi, -\tfrac{1}{\sqrt{6}}b_\eta + \tfrac{1}{\sqrt{2}}b_\pi\right), \\ B^{(1)} &= \mathrm{diag}\left(\tfrac{1}{\sqrt{3}}b_\sigma, \tfrac{1}{\sqrt{3}}b_\sigma, \tfrac{1}{\sqrt{3}}b_\sigma\right), \end{aligned} \tag{3.1}$$

which behave as if those were octet and singlet of SU(3), respectively, where

$$b_\eta = \sqrt{2}\sin\beta, \quad b_\pi = \sqrt{2}\cos\beta, \quad b_\sigma = 1, \tag{3.2}$$

and the factor $\sqrt{2}$ in Eq.(3.2) has been chosen as $b_\pi^2 + b_\eta^2 = 2$ compared with $b_\sigma^2 = 1$. Then, we can express the symmetry breaking term as the form

$$W_{SB} = \frac{\sqrt{3}}{2}\varepsilon m\left[\mathrm{Tr}(B^{(8)}\phi^{(8)}\phi^{(8)}) + \mathrm{Tr}(B^{(1)}\phi^{(1)}\phi^{(1)})\right]$$

$$= \frac{1}{2}\varepsilon m \left[-2\phi_\pi\phi_\eta \cos\beta - (\phi_\pi^2 - \phi_\eta^2)\sin\beta + \phi_\sigma^2\right], \tag{3.3}$$

where the factor $\sqrt{3}/2$ has been chosen as the coefficients in the expression (3.3) correspond to those in the unbroken form (1.8). Although the term $\mathrm{Tr}(B^{(1)}\phi^{(1)}\phi^{(1)}) = \phi_\sigma^2/\sqrt{3}$ in (3.3) does not break the S_4 symmetry, it has been added by hand in order that the term W_{SB} (in other words, the parameter ε) does not affect the VEV relation (1.5).

As the result, we can write the superpotential including the symmetry breaking term as follows:

$$W = \frac{1}{2}m\left\{\phi_\pi^2 + \phi_\eta^2 + (1+\varepsilon)\phi_\sigma^2 - \varepsilon\left[2\phi_\pi\phi_\eta\cos\beta + (\phi_\pi^2 - \phi_\eta^2)\sin\beta\right]\right\} + \frac{1}{2}\lambda\phi_\sigma\left(\phi_\eta^2 + \phi_\pi^2 + \frac{1}{3}\phi_\sigma^2\right). \tag{3.4}$$

Since

$$\frac{\partial W}{\partial \phi_\pi} = \left[m + \lambda\phi_\sigma - \varepsilon m \sin\beta\right]\phi_\pi - \varepsilon m \phi_\eta \cos\beta, \tag{3.5}$$

$$\frac{\partial W}{\partial \phi_\eta} = \left[m + \lambda\phi_\sigma + \varepsilon m \sin\beta\right]\phi_\eta - \varepsilon m \phi_\pi \cos\beta, \tag{3.6}$$

$$\frac{\partial W}{\partial \phi_\sigma} = m(1+\varepsilon)\phi_\sigma + \frac{1}{2}\lambda(\phi_\pi^2 + \phi_\eta^2 + \phi_\sigma^2), \tag{3.7}$$

the minimizing conditions of the potential leads to the relations

$$\tan\beta = \frac{v_\pi^2 - v_\eta^2}{2v_\pi v_\eta}, \tag{3.8}$$

$$v_\pi^2 + v_\eta^2 = v_\sigma^2, \tag{3.9}$$

$$m(1+\varepsilon) + \lambda v_\sigma = 0. \tag{3.10}$$

Note that the derivation of the relation (3.8) is independent of the explicit values of m, λ and ε, and the derivation of the relation (3.9) is independent of the explicit values of m, λ, ε and β. Thus, we can fix the value of v_π/v_η by the parameter β in W_{SB} without spoiling the VEV relation (1.5) [(3.9)]. Also note that the limit $m_e \to 0$ corresponds to the limit $v_\eta \to -v_\sigma/\sqrt{2}$ (i.e. $v_\eta^2 = v_\pi^2$), so that the limit $m_e \to 0$ corresponds to $\beta \to 0$.

When we define the parameters $z_i = \sqrt{m_{ei}}/\sqrt{m_e + m_\mu + m_\tau}$, from the observed values [1] of the charged lepton masses, we obtain the numerical values $z_1 = 0.016473$, $z_2 = 0.236869$ and $z_3 = 0.971402$, so that, for the VEVs of ϕ_a defined by Eq.(1.3) [(2.2)], we obtain $z_\pi = 0.519393$, $z_\eta = -0.479824$ and $z_\sigma = 1/\sqrt{2} = 0.707106$. Therefore, we can estimate the value of β as follows:

$$\sin\beta = \frac{z_\eta^2 - z_\pi^2}{z_\eta^2 + z_\pi^2} = 4z_\eta^2 - 1 = -0.079078, \qquad \beta = -4.5355^\circ, \tag{3.11}$$

where we have chosen the phase convention of β as $\cos\beta = -2z_\pi z_\eta/(z_\eta^2 + z_\pi^2) > 0$.

Table 1 SU(3) and S_4 assignments of the fields

Fields	$SU(2)_L$	SU(3)	S_4	Z_3	Z_3'	Z_2
ℓ_L	$\mathbf{2}$	$\mathbf{3}$	$\mathbf{3}'$	0	0	0
e_R	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}'$	0	0	0
$\nu_R^{(\pm)}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	0	0/+1
ϕ_u	$\mathbf{1}$	$\mathbf{1}$+$\mathbf{8}$	$\mathbf{1}+(\mathbf{2}+\mathbf{3}+\mathbf{3}')$	+1	+1	0/+1
ϕ_d	$\mathbf{1}$	$\mathbf{1}$+$\mathbf{8}$	$\mathbf{1}+(\mathbf{2}+\mathbf{3}+\mathbf{3}')$	-1	-1	0/+1
$\xi^{(\pm)}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	-1	0/+1
χ	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}'$	+1	-1	0
H_L^u	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	+1	0	0
H_L^d	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	-1	0	0
Φ_R	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	0	0

From the point of view of the prameter physics, the introducing the symmetry breaking term (3.3) is merely replacing the parameter v_π/v_η by another parameter β. What is important is that we can indeed introduce a symmetry breaking term without spoiling the relation (1.5).

4 Effective Hamiltonian

If we regard the scalars ϕ_u and ϕ_d as $SU(2)_L$ doublets, such a model with multi-Higgs doublets causes a flavor changing neutral current (FCNC) problem. Therefore, we must consider that the fields ϕ_u and ϕ_d are $SU(2)_L$ singlets. In the present paper, we assume a Froggatt-Nielsen [12] type model

$$H^{eff} = y_e \bar{\ell}_L H_L^d \frac{\phi_d}{\Lambda}\frac{\phi_d}{\Lambda}\frac{\xi}{\Lambda} e_R + y_\nu \bar{\ell}_L H_L^u \frac{\phi_u}{\Lambda}\frac{\chi}{\Lambda}\nu_R + y_R \overline{\nu}_R \Phi_R \nu_R^*, \tag{4.1}$$

where ℓ_{iL} are $SU(2)_L$ doublet leptons $\ell_{iL} = (\nu_{iL}, e_{iL})$, H_L^d and H_L^u are conventional $SU(2)_L$ doublet Higgs scalars, ϕ_f $(f = u, d)$, ξ and χ are $SU(2)_L$ singlet scalars, and Λ is a scale of the effective theory. We consider that $\langle\phi_f\rangle/\Lambda$, $\langle\xi\rangle/\Lambda$ and $\langle\chi\rangle/\Lambda$ are of the order of 1. The scalar Φ_R has been introduced in order to generate the Majorana mass M_R of the right-handed neutrino ν_R. As we note later, in the present model, the right-handed neutrinos $\nu_R = (\nu_R^{(+)} + \nu_R^{(-)})/\sqrt{2}$ are singlets of the SU(3) flavor. The role of $\xi = (\xi^{(+)} + \xi^{(-)})/\sqrt{2}$ and χ will be explained later. In order to understand the appearance of the combinations $H_L^d \phi_d \phi_d \xi$ and $H_L^u \phi_u \chi$, we assume two Z_3 symmetries (Z_3 and Z_3' in Table 1). Those quantum number assignments are given in Table 1. However, even with those quantum numbers, we cannot distinguish the state $\phi_f^\dagger$ from $\phi_f\phi_f$. For example, the interaction $\bar{\ell}_L H_d \phi_d^\dagger \xi e_R$ is possible in addition to the interaction $\bar{\ell}_L H_d \phi_d \phi_d \xi e_R$. Although we have started from an SUSY senario in the previous section, now, we have adopted an effective Hamiltonian which is not renormalizable. Therefore, in principle, the interaction $\bar{\ell}_L H_d \phi_d^\dagger \xi e_R$ cannot be ruled out. For the moment, in order to forbid such an undesirable term, we assume that the fields which can appear in the effective Hamiltonian are confined to holomorphic ones.

(a) Charged lepton sector

Recall that we have already assumed the invariance of the superpotential under the Z_2 transformation (2.5) in order to drop the cubic part of the octet $\phi^{(8)}$. Therefore, the term $\phi\phi$ means $\phi^{(8)}\phi^{(8)} + \phi^{(1)}\phi^{(1)}$ under the Z_2 invariance. However, in order to give $m_{ei} \propto \langle\phi_i^i\rangle^2$, what we want is not $\phi^{(8)}\phi^{(8)} + \phi^{(1)}\phi^{(1)}$, but $\phi^{(8)}\phi^{(8)} + \phi^{(1)}\phi^{(1)} + \phi^{(8)}\phi^{(1)} + \phi^{(1)}\phi^{(8)}$. In order to evade this problem, we introduce additional fields $\xi^{(+)}$ and $\xi^{(-)}$ whose Z_2 parity are $+1$ and -1, respectively. The effective interactions in the charged lepton sector are given by

$$H_e^{eff} = \frac{y_e}{\sqrt{2}}\bar{e}_L^i(\phi_d)_i^j(\phi_d)_j^k(\xi^{(+)} + \xi^{(-)})e_{Rk}, \tag{4.2}$$

where we have dropped the Higgs scalar H_L^d since we discuss flavor structure only. The expression (4.2) becomes

$$H_e^{eff} = \frac{y_e}{\sqrt{2}}\bar{e}_L[(\phi_d^{(8)}\phi_d^{(8)} + \phi_d^{(1)}\phi_d^{(1)})\xi^{(+)} + (\phi_d^{(8)}\phi_d^{(1)} + \phi_d^{(1)}\phi_d^{(8)})\xi^{(-)}]e_R. \tag{4.3}$$

Since we have assumed that $\xi^{(+)}$ and $\xi^{(-)}$ appear symmetrically in the theory, we also assume

$$\langle\xi^{(+)}\rangle = \langle\xi^{(-)}\rangle \equiv v_\xi. \tag{4.4}$$

Then, we obtain the effective Hamiltonian for the charged leptons

$$H_e^{eff} = \frac{y_e v_d v_\xi}{\sqrt{2}\Lambda^3}\sum_i \bar{e}_L^i\langle(\phi_d^{(8+1)})_i^i\rangle^2 e_{Ri}, \tag{4.5}$$

where $v_d = \langle H_L^{d0}\rangle$. Since the fields $(\phi_d)_i^i$ are defined by Eq.(2.2), we can obtain the charged lepton mass relation (1.1) from the VEV relation (1.6).

However, the present mechanism to obtain $m_{ei} \propto \langle\phi_i^i\rangle^2$ is somewhat artificial. The present mechanism will be improved in the future model. [Of course, there is a possibility that the superpotential (2.3) must exactly be invariance under the Z_2 symmetry, but the effective Hamiltonian (4.1) does not need to be invariance under the Z_2 symmetry. Then, we can consider a model without $\xi^{(\pm)}$.]

(b) Neutrino sector

In the present model, the right-handed neutrinos $\nu^{(\pm)}$ are singlets of SU(3). Therefore, in the neutrino seesaw mass matrix $M_\nu = m_L^\nu M_R^{-1}(m_L^\nu)^T$, M_R is a 1×1 matrix and m_L^ν is a 3×1 matrix. In order to compensate for the absence of the conventional triplet neutrinos ν_R, a new scalar χ which is a triplet of SU(3) has been introduced. The neutrino Dirac mass terms are given by the following effective Hamiltonian

$$H_{Dirac}^{eff} = y_\nu\frac{v_u}{\Lambda^2}\bar{\nu}_L^i\langle(\phi_u)_i^j\rangle\langle\chi_j\rangle(\nu_R^{(+)} + \nu_R^{(-)}), \tag{4.6}$$

where $v_u = \langle H_L^{u0}\rangle$. It is likely that the scalar potential $V(\chi)$ for the SU(3) triplet χ has a specific VEV solution

$$\langle\chi_1\rangle = \langle\chi_2\rangle = \langle\chi_3\rangle \equiv v_\chi. \tag{4.7}$$

When we assume the VEVs (4.7), we obtain

$$H_{Dirac}^{eff} = y_\nu \frac{v_u v_\chi}{\sqrt{2}\Lambda^2} (\bar{\nu}_\eta \ \bar{\nu}_\sigma \ \bar{\nu}_\pi)_L \left[\begin{pmatrix} v_\eta \\ 0 \\ v_\pi \end{pmatrix} \nu_R^{(-)} + \begin{pmatrix} 0 \\ v_\sigma \\ 0 \end{pmatrix} \nu_R^{(+)} \right], \tag{4.8}$$

where $v_a = \langle \phi_{ua} \rangle$ $(a = \pi, \eta, \sigma)$ (for convenience, we have dropped the index u). Therefore, we obtain the effective neutrino mass matrix on the (η, σ, π) basis,

$$U_{TB}^T M_\nu U_{TB} \equiv M_\nu^{(\eta\sigma\pi)} = \frac{1}{M_R^{(-)}} \begin{pmatrix} v_\eta^2 & 0 & v_\pi v_\eta \\ 0 & 0 & 0 \\ v_\pi v_\eta & 0 & v_\pi^2 \end{pmatrix} + \frac{1}{M_R^{(+)}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & v_\sigma^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{4.9}$$

where $M_R^{(\pm)} = y_R^{(\pm)} \langle \Phi_R \rangle$, and we have dropped the common factors $(y_\nu v_u v_\chi / \sqrt{2}\Lambda^2)^2$. By the way, the ratio v_π / v_η cannot be determined from the potential (2.6), and the ratio is determined by a soft S_4 symmetry breaking term W_{SB} which has been discussed in the previous section. We can choose a solution $v_\pi = 0$ in the superpotential $W(\phi_u)$ by adjusting the parameter β in W_{SB}, differently from the case of $W(\phi_d)$. Then, the neutrino mass matrix (4.9) becomes a diagonal form $D_\nu = (1/M_R^{(-)})\mathrm{diag}(v_\eta^2, 0, 0) + (1/M_R^{(+)})\mathrm{diag}(0, v_\sigma^2, 0)$. Since the mass matrix M_ν on the $(\nu_1, \nu_2, \nu_3) = (\nu_e, \nu_\mu, \nu_\tau)$ basis is given by

$$M_\nu = U_{TB} M_\nu^{(\eta\sigma\pi)} U_{TB}^T = U_{TB} D_\nu U_{TB}^T, \tag{4.10}$$

we can obtain the tribimaximal mixing

$$U_\nu = U_{TB}, \tag{4.11}$$

and the neutrino masses

$$m_{\nu 1} = k v_\eta^2, \quad m_{\nu 2} = k v_\sigma^2, \quad m_{\nu 3} = 0, \tag{4.12}$$

for the case of $M_R^{(+)} = M_R^{(-)} \equiv M_R$, where $k = (y_\nu v_u v_\chi)^2 / 2 M_R \Lambda^4$ and $(\nu_\eta, \nu_\sigma, \nu_\pi)$ has been renamed (ν_1, ν_2, ν_3) according to the conventional naming.

However, since we have taken $v_\pi = 0$, the value of v_η satisfies $v_\eta^2 = v_\sigma^2$ from the relation (1.5), so that the result (4.12) gives $m_{\nu 1} = m_{\nu 2}$. The observed value [14] Δm_{solar}^2 is small, but it is not zero. Therefore, we must consider a small deviation between the first and second terms in (4.9) (i.e. $M_R^{(+)} \neq M_R^{(-)}$). Since the value $M_R^{(-)}/M_R^{(+)}$ is free in the present model, we cannot predict an explicit value of the ratio $\Delta m_{solar}^2 / \Delta m_{atm}^2$.

Since the present model gives an inverse hierarchy of the neutrino masses, the predicted effective electron neutrino mass

$$\langle m_{\nu_e} \rangle = \left| \sum_i U_{ei}^2 m_{\nu i} \right| \simeq |m_{\nu 1}| \simeq |m_{\nu 2}| \simeq \sqrt{\Delta m_{atm}^2} = 5.23_{-0.40}^{+0.25} \times 10^{-2}\,\mathrm{eV}, \tag{4.13}$$

where we have used the value [15] $\Delta m^2_{atm} = 2.74^{+0.44}_{-0.26} \times 10^{-3}\,\mathrm{eV}^2$. This value (4.13) is sufficiently sensitive to the next generation experiments of the neutrinoless double beta decay.

5 Summary

In conclusion, on the basis of the S_4 symmetry which is embedded into SU(3), we have investigated a lepton mass model with the effectuve Hamiltonian of the Froggatt-Nielsen type (4.1). We have assumed that the singlet and doublet of S_4 originate in the singlet and octet of SU(3), and we have obtained the VEV relation (1.5). In the derivation of the VEV relation (1.5), the essential assumptions for the superpotential $W(\phi_f)$ are the following two: (i) the scalar fields ϕ_f always appear in terms of the nonet form (2.1) of U(3); (ii) the superpotential $W(\phi_f)$ is invariant under the Z_2 transformation (2.5). Then, we have obtained not only the VEV relation (1.5), but also $\langle(\phi^{(8)})^j_i\rangle = 0$ $(i \neq j)$ for the other components of $\phi^{(8)}$ (i.e. $\langle \mathbf{3} \rangle = \langle \mathbf{3'} \rangle = 0$).

In the charged lepton secter, in order to give $m_{ei} \propto \langle(\phi_d)^i_i\rangle^2$, we have assumed new scalars $\xi^{(\pm)}$. Although it has been reuired to compensate for the Z_2 invariance, the model seems to leave the door open to further improvement.

For the neutrino sector, we have obtained the tribimaximal mixing (1.2) by introducing an SU(3) triplet scalar χ and the two SU(3) singlet right-handed neutrinos $\nu_R^{(\pm)}$ in addition to the nonet scalar ϕ_u. In the present model, the right-handed neutrinos $\nu_R^{(\pm)}$ are singlets of SU(3), the Majorana neutrino mass matrices $M_R^{(\pm)}$ have no flavor structure. For the neutrino mass spectrum, since the model gives $m_{\nu 1} = m_{\nu 2}$ in the limit of $M_R^{(+)} = M_R^{(-)}$, we must consider a small deviation $M_R^{(+)} \neq M_R^{(-)}$. Since the value of $M_R^{(-)}/M_R^{(+)}$ is a free parameter in the present model, we cannot predict the value $\Delta m^2_{solar}/\Delta m^2_{atm}$ at present, although the smallness of the ratio $\Delta m^2_{solar}/\Delta m^2_{atm}$ can be understood. Since the present model gives an inverse hierarchy of the neutrino masses, we can predict the effective electron neutrino mass $\langle m_{\nu_e}\rangle \simeq 0.05$ eV, which is sufficiently sensitive to the next generation experiments of the neutrinoless double beta decay.

The present model seems to provide suggestive hints on seeking for a model which leads to the tribimaximal mixing (1.2) and the charged lepton mass relation (1.1), although the model has still many points which should be improved. At the same time, the model will provide a clue to the quark mass matrix model from a point of unified view of the quarks and leptons. The extension of the present model to the quark mass matrix model will be given elsewhere.

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